Uninsurable Risk and the Determination of Real Interest Rates: An Investigation using UK Indexed Bonds

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Abstract

This paper investigates the empirical performance of a new class of consumption-based, uninsurable risk models in the context of UK indexed bond market. Using closed form expressions for pricing kernels, we test the ability of three consumption-based models to fit the market prices of indexed bonds in the UK. The classical general equilibrium model performs reasonably well, in contrast to its performance in equity-pricing exercises, but is marginally outperformed by models that limit the availability of insurance. A model that prohibits all insurance appears to perform marginally better than a model that permits partial insurance. In contrast to the estimates that typically arise in equity markets, the estimated coefficient of relative risk aversion, and the resulting bond risk premia. are found to be small in the indexed bond market. The estimated price equations are used to calculate impulse responses illustrate the effects of various macroeconomic shocks on real interest rates.

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1 Introduction

The influence of uninsurable risk on financial risk premia has generated increasing interest among financial economists in recent years. Among the most recent papers, Kocherlakota and Pistaferri (2007, 2009) look at two uninsurable-risk settings: (i) an incomplete market environment (INC henceforth) in which idiosyncratic consumption risks are entirely uninsurable, and (ii) an alternative market environment in which these private risks are insurable but only partly so, due to incentive constraints on agents' truth revelation about their private shocks. Kocherlakota and Pistaferri label this market structure 'private information pareto optimal' (PIPO), since its consumption allocation is constrained Pareto optimal. They also demonstrate that the PIPO structure performs much better in terms of explaining the equity premium and real exchange rate puzzles than the representative agent complete market and INC models.

The superior performance of the PIPO model is also apparent in Basu, Semenov and Wada (2007), in which the same three models are used in an attempt to reconcile the equity premium, international risk sharing and currency premium puzzles. Part of the purpose of this paper is to investigate the possibility that this superior performance also arises in the relatively simple context of real interest rates and indexed bonds.

In this paper we test the ability of consumption-based models to price a combined cross-section and time-series of UK inflation-indexed government bonds. Indexed, or 'real', bonds are an attractive test bed because their prices provide the real risk free rate, which is common to the discount rates of all other assets. Consequently, if a model cannot perform well in this market it is reasonable to expect that it will face difficulties in other markets as well.[1] In addition, one can also undertake useful investigation about how the implied real interest rates depend on economic fundametals.

Our paper makes three principal contributions. First, it models the market prices of indexed bonds in a general equilibrium setting with uninsurable risks, and is, to the best of our knowledge, the first paper to do so. Second, we derive closed form expressions for "indexed" bond prices based on uninsurable risk as measured by the cross sectional, time-varying, variance

[1] In practice, indexed bonds are not quite 'real' bonds since there remains a small inflation risk due to unavoidable imperfections in the indexation process. In our model, this inflation risk interacts with the uninsurable consumption risk in determining the bond risk premia.

of consumption. Finally, using this indexed-bond price formula we derive implied real risk premia, and investigate the response of real rates to changes in the model's state variables.

An important aspect of this investigation is that we fit the models to the market prices of the bonds directly, rather than to estimates of real interest rates constructed from the bond prices, such as those provided by the Bank of England. As a result, our estimates are not influenced by the filtering methods used in the construction of these real rates.

The cornerstone of our model is a closed form lognormal bond price equation in which expected future values of the explanatory variables are constructed from a vector autoregression. As explanatory variables, or factors, we choose two macroeconomic fundamentals that come directly from the underlying equilibrium model: aggregate consumption growth, and the growth rate of the cross sectional log variance of consumption (which represents uninsurable risk). A third factor, inflation, is included because we are fitting the market prices of indexed bonds, which can be shown to depend on expected inflation due to the imperfect nature of their indexation. First, a parsimonious VAR is estimated for the three factors and then, using these VAR estimates, the structural parameters, i.e. agent's risk aversion and pure rate of time preference, are estimated using the bond price data and the closed form bond pricing equation. This approach allows us to investigate *ex-ante* risk premia in real rates by computing the implied premia for coupon bonds of various maturities, and to compute the impulse responses of real interest rates with respect to our three macroeconomic fundamentals.

Our results suggest that the standard complete markets model performs quite well when allowed to focus on real rates rather than on equity returns. The models with uninsurable risk also perform well, and in some respects outperform the standard model. The estimates of the coefficient of risk aversion and the resulting bond risk premia are found to be small but these are consistent with the low returns earned on UK indexed bonds during our sample period. The impulse response analysis with the estimated bond price equations reveals that a rise in inflation lowers real interest rates of nearly all maturities. A rise in consumption growth raises real interest rates as suggested by the permanent income hypothesis. Greater uninsurable consumption risk, on the other hand, lowers the real interest rates driven by a precautionary saving motives.

The paper is organized as follows. The following section lays out the basic setup for the three pricing kernels. Section 3 presents the applications of these pricing kernels to UK "indexed" bond prices. Section 4 discusses the estimation methods and the data. Section 5 presents the estimation results. Section 6 ends with concluding comments.

2 Basic Setup

2.1 Three Pricing Kernels

Our benchmark case is the traditional complete market model with homogeneous agents. With a power utility function (with risk aversion parameter γ), the stochastic discount factor is given by:

$$M_{t+1}^{RA} = \frac{\beta c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \tag{1}$$

where β is the subjective discount factor and c_t is the aggregate consumption at date t.

In two influential papers (2007, 2009) Kocherlakota and Pistaferri (K-P hereafter) introduce consumer heterogeneity and uninsurable risk for two distinct market environments: (i) incomplete market (INC) where private skill shocks are uninsurable, (ii) partial insurance environment where the private skill shocks are partially insured by an insurance company who stipulate long term contracts with agents subject to a truth revelation constraint for eliciting efforts and private skill shocks. The latter environment is constrained Pareto efficient and K-P call it private information Pareto optimal ($PIPO$) environment.

Using the law of large numbers K-P demonstrate that the pricing kernels for these two market environments can be written as:

$$M_{t+1}^{INC} = \frac{\beta C_{-\gamma,t+1}}{C_{-\gamma,t}} \tag{2}$$

$$M_{t+1}^{PIPO} = \frac{\beta C_{\gamma,t}}{C_{\gamma,t+1}} \tag{3}$$

where $C_{-\gamma,t}$ is the nagtive γth cross sectional raw moment of consumption at date t and likewise $C_{\gamma,t}$ is positive counterpart of the same cross sectional moment.

2.2 Lognormal Parameterization of Consumption Processes

We consider a lognormal parameterization of the post-trade consumption process. Following Sarkissian (2003) we represent the *post-trade* allocation of consumption as follows.[2] The i^{th}t investor's consumption is:

$$c_{i,t} = \delta_{i,t}.c_t \tag{4}$$

where $\delta_{i,t}$ is the ith investor's share in aggregate consumption , c_t. We assume the following lognormal process for $\delta_{i,t}$:

$$\delta_{i,t} = \exp(u_{i,t}\sqrt{x_t} - \frac{x_t}{2}) \tag{5}$$

where $u_{i,t}$ is standard normal i.i.d shock, and x_t is the cross sectional variance of log consumption.

The s^{th} raw moment of the cross sectional distribution of consumption is given by:

$$E_i\left(c_i^s\right) = c_t^s \exp\left(\frac{(s^2 - s)}{2} x_t\right) \tag{6}$$

Note that, by construction, aggregate consumption is the sum of individual consumption, which can be checked by setting $s = 1$. We now address the issue of whether or not it satisfies the optimality conditions. We follow the same reverse engineering approach as in Basu, Semenov and Wada (2007): If there is a unique pricing kernel that supports this allocation of consumption, then it must also support the individual optimality conditions.

[2] Sarkissian (2003) writes the post trade allocation in terms of the consumption growth rate while we write it here in terms of the level of consumption. The motivation for doing this is to allow us to apply this post-trade allocation to the Kocherlakota-Pistaferri (2007, 2009) discounting methodology. The Kocherlakota-Pistaferri incomplete market discount factor is based on the growth rates of the cross sectional moments of the level of consumption while Sarkissian (2003) and Basu and Wada (2006) use the Constantinides-Duffie (1996) discount factor which is based on the cross sectional average of the intertemporal marginal rates of substitution.

2.3 Lognormal Pricing Kernels

Substituting (6) into (2), and evaluating at $s = -\gamma$, gives the following pricing kernel for the INC environment :

$$M_{t+1}^{INC} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \exp \left(\frac{\gamma(\gamma+1)}{2} \left(x_{t+1} - x_t \right) \right) \tag{7}$$

Similarly, substitution of (6) into (3), and evaluating at $s = \gamma$, gives:

$$M_{t+1}^{PIPO} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \exp \left(\frac{-\gamma(\gamma-1)}{2} \left(x_{t+1} - x_t \right) \right) \tag{8}$$

3 Application to UK Indexed Bonds

3.1 Pricing pure-real zero-coupon bonds

We start by considering the real price, P_{nt}^R, of a zero-coupon bond with maturity n, and develop this into the nominal price of the imperfectly indexed coupon bonds that are traded in the UK. P_{nt}^R can be written as follows for each of the market environments, $h = RA, INC, PIPO$:

$$P_{nt}^R = E_t \left[P_{n-1,t+1}^R M_{t+1}^h \right] \tag{9}$$

Assuming lognormality we get the following expression for the log real price of a perfectly indexed zero-coupon bond of maturity n,

$$p_{nt}^R = E_t[m_{t+1}^h + p_{n-1,t+1}^R] + \frac{1}{2} Var_t[m_{t+1}^h + p_{n-1,t+1}^R] \tag{10}$$

where

$$m_{t+1}^{RA} = ln(\beta) - \gamma g_{t+1} \tag{11}$$

$$m_{t+1}^{INC} = ln(\beta) - \gamma g_{t+1} + \left(\frac{\gamma(\gamma+1)}{2} \right) v_{t+1} \tag{12}$$

$$m_{t+1}^{PIPO} = ln(\beta) - \gamma g_{t+1} - \left(\frac{\gamma(\gamma-1)}{2} \right) v_{t+1} \tag{13}$$

and $g_{t+1} \equiv c_{t+1} - c_t$, $v_{t+1} \equiv x_{t+1} - x_t$.[3]

[3] In K-P's (2007, 2009) setup, there are both aggregate and individual shocks and

3.2 Pricing imperfectly indexed coupon bonds

UK indexed bonds are indexed to the change in goods prices[4] over a base period starting 8 months before their issue date, and ending 8 months before their redemption date.[5] We approximate this eight-month lag by 3 calendar quarters because we are using quarterly data. Thus the total inflation compensation for an n-period zero-coupon indexed bond is Q_{t+n-3}/Q^*, where Q^* is the goods price for the bond's base period, which leaves the bond's real price exposed to inflation over final 3 periods of its life. Thus equation (9) becomes

$$P_{nt}^{R} = E_t\left[\left(\prod_{s=1}^{n} M_{t+s}^{h}\right)\frac{Q_{t+n-3}}{Q^*}\frac{1}{Q_{t+n}}\right] \tag{14}$$

from which we get the nominal price of the bond as,

$$P_{nt}^{Nom} = \frac{Q_t}{Q^*}E_t\left[\left(\prod_{s=1}^{n} M_{t+s}^{h}\right)\frac{Q_{t+n-3}}{Q_{t+n}}\right] \tag{15}$$

Using lognormality and denoting the lower cases as the log of upper cases, (15) can written as:

$$p_{nt}^{Nom} = (q_t - q^*) + E_t[z_{n,t+1}] + \frac{1}{2}Var_t[z_{n,t+1}] \tag{16}$$

where

$$z_{n,t+1} = \sum_{s=1}^{n} m_{t+s} - \sum_{s=0}^{2} \pi_{t+n-s} \tag{17}$$

and $\pi_{t+s} = q_{t+s} - q_{t+s-1}$.

The nominal price, in natural units, of a bond that pays a quarterly

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the former are completely hedged by a set of aggregate-shock contingent claims. In our bond economy, if these contingent claims do not exist in addition to bonds, PIPO market environment is not constrained pareto optimal and the use of the PIPO discount factor is not justified. In order to avoid this problem, we assume that both bonds, and these contingent claims, are traded. For reasons of brevity we focus here on bonds. A fully specified model is, however, available on request from the authors.

[4] Measured by the Retail Prices Index (RPI).

[5] The indexation method for UK bonds changed in 2005 (after the end of our sample). For bonds issued since that date the indexation lag is 3 months.

coupon[6] C can then be expressed as a linear combination of zero coupon log prices as follows:

$$P_{nt}^{Nom,c} = \sum_{s=1}^{n} exp(p_{st}^{Nom})C + exp(p_{nt}^{Nom}) \tag{18}$$

This price is exposed to changes in current inflation to the extent that it influences expectations of future inflation and the consumption components of the stochastic discount factor.

4 Estimation Method and Data

Our focus is on maximum likelihood estimation of the lognormal bond pricing models described above. We use a 'panel' of observed prices consisting of a time-series of a selection of about six bonds in each period. The structural parameters can be estimated from a single cross-section, or from a time-series of prices for a single bond. Subject to parameter stability, the simultaneous use of both cross-sectional and time-series data should increase the efficiency of the estimates and provide a sharper test of the model than we get from either cross-section or time-series estimation alone.

4.1 Maximum Likelihood Estimation

4.2 A vector autoregressive model for the state variables

The nominal coupon bond price $P_{nt}^{Nom,c}$ in (18) through (16) depends on expectations of the three state variables; consumption growth (g), the change in the cross-sectional variance of consumption (v), and inflation (π), which we generate from a separately estimated vector autoregression as explained below.

[6] UK indexed coupons are paid 6-monthly. We fit this into our quarterly model by assuming half of the 6-monthly coupon to be paid each quarter. This introduces a small error due to the overvaluation of each coupon that accompanies our assumption that half of it is paid earlier than it is in reality.

Let w_t be a vector of state variables

$$w_t = \begin{pmatrix} g_t \\ v_t \\ \pi_t \end{pmatrix} \tag{19}$$

We assume the state vector to be autoregressive

$$w_{t+1} = A + Bw_t + \epsilon_{t+1} \tag{20}$$

where

$$\epsilon_{t+1} \sim N(0, \Omega_\epsilon) \qquad \forall t$$

We define a set of coefficient vectors ϕ_R to be consistent with equations (11) to (13) as follows:

ϕ_R^{RA}	ϕ_R^{INC}	ϕ_R^{PIPO}
$-\gamma$	$-\gamma$	$-\gamma$
0	$\frac{\gamma(\gamma+1)}{2}$	$\frac{-\gamma(\gamma-1)}{2}$
0	0	0

and define a second set ϕ_L, to capture the effects of inflation, as

$$\phi_L = \phi_R + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

The log of the pricing kernels can then be written as,

$$m^h_{n,t+1} = ln(\beta) + \phi'_R w_{t+1} \tag{21}$$

and, from (17),

$$E_t[z_{n,t+1}] = \sum_{i=1}^{n-3} \phi'_R E_t[w_{t+i}] + \sum_{i=n-2}^{n} \phi'_L E_t[w_{t+i}] + ln(\beta) \tag{22}$$

$$Var_t[z_{n,t+1}] = \sum_{i=1}^{n-3} \phi'_R \Omega_{t+i} \phi_R + \sum_{i=n-2}^{n} \phi'_L \Omega_{t+i} \phi_L \tag{23}$$

where

$$
\begin{aligned}
E_t[w_{t+i}] &= \tilde{B}_i A + B w_t && (24)\\
\Omega_{t+i} &= \sum_{j=0}^{i-1} B^j \Omega_\epsilon B^{j'} \quad \forall\ t && (25)
\end{aligned}
$$

and

$$\tilde{B}_i = \sum_{j=0}^{i-1} B^j$$

After substituting (22) and (23) into (16), the real price of the indexed zero-coupon bond can then be expressed in familiar affine form as:

$$p_{nt}^R = G_n + H_n w_t \tag{26}$$

where

$$
\begin{aligned}
G_n &= ln(\beta) + \left(\sum_{i=1}^{n-3} \phi'_R \tilde{B}_i A + \sum_{i=n-2}^{n} \phi'_L \tilde{B}_i A \right) + \\
&\quad \frac{1}{2} \left(\sum_{i=1}^{n-3} \sum_{j=0}^{i-1} \left(\phi'_R B^j \Omega_\epsilon B^{j'} \phi_R \right) + \sum_{i=n-2}^{n} \sum_{j=0}^{i-1} \left(\phi'_L B^j \Omega_{t+i} B^{j'} \phi_L \right) \right) && (27)\\
H_n &= \sum_{i=1}^{n-3} \phi'_R B + \sum_{i=n-2}^{n} \phi'_L B && (28)
\end{aligned}
$$

The log nominal price follows as $p_{nt}^{Nom} = p_{nt}^R + q_t$ which we substitute into (18) to obtain our estimation equation.[7]

$$P_{nt}^{Nom,c} = \sum_{s=1}^{n} exp(p_{st}^R + q_t)C + exp(p_{nt}^R + q_t) \tag{29}$$

We first estimate the vector autoregression for the state variables in order to obtain estimates of A, B and Ω_ϵ, and then use maximum likelihood to estimate the parameters (β and γ) of the asset pricing models by fitting equation (29) to market prices. The pricing errors are assumed to be normally

[7] The details of the derivation are presented in Appendix B.

and independently distributed, and homoskedastic across both maturities and time.

Using all of the available data in this way greatly increases the number of degrees of freedom, but does so at the cost of imposing parameter constancy over the sample. Some degree of persistence in the parameter values seems reasonable, so our approach offers a potential efficiency gain over the familiar approach of estimating the yield curve parameters for each period independently. To allow for the possibility that the parameters change with changes in the policy regime we also estimate the model over a number of sub samples, as discussed below.

4.3 Data

We use bond price data from the UK Debt Management Office. Since all indexed bonds with a maturity of 8 months or less, are pure nominal bonds we select only bonds with a residual maturity of 2 years or more. The number of indexed bonds in the market in any quarter is very small, ranging from 7 to 9. We select 6 bonds in each period, aiming for as even a spread as possible across the maturities from 1 to 25 years. When choosing between bonds with similar maturities, we select the one with the largest issue size.

Aggregate real consumption data are from the Office for National Statistics, and the cross sectional variances of the log of real consumption are from the Family Expenditure Survey (FES).[8] Data are quarterly for the period 1983Q1 to 2004Q4 and are seasonally unadjusted.[9]

4.4 Sub-samples and Monetary Policy Regimes

We estimate the model over the full sample 1983Q2 to 2004Q4, and over the following sub-samples:

[8]The FES was replaced by the Expenditure and Food Survey, which also covered the National Food Survey, in April 2001.

[9]The details of the computation of the cross sectional variances are presented in Appendix A.

Sub sample	Monetary policy regime
1983Q2 to 1990Q3	Monetary growth and exchange rate targets.
1990Q3 (Oct) to 1992Q3 (Sept)	Exchange Rate Mechanism.
1992Q4 to 1997Q2	Inflation target.
1997Q3 to 2004Q4	Inflation target with Bank of England independence.

From 1983 to 1992 the UK sought to anchor inflation first with control of monetary aggregates, then by using an informal combination of monetary and exchange rate targets, and finally with a 2-year membership of the European Exchange Rate Mechanism (ERM). In the post-ERM period inflation was targeted directly by the Treasury, and then, from 1997, by the newly independent Bank of England.

4.5 Policy regimes and dummy variables.

The pattern of real interest rates that appears in the Bank of England's own estimates[10] reflects the policy regimes noted in the previous section. There is, however, no such pattern in the consumption variables in our estimated model, which suggests that, even if real rates are influenced by the consumption variables, there are important policy-related influences present also. It is also clear that, for our sample period, these influences had an important impact on real rates. Ideally, the policy-related effects should be brought within the asset pricing model, but we leave this step to future research and focus here on the role of the consumption variables.

In order to address the question of whether or not the consumption variables make a statistically significant contribution to the determination of real rates we add a dummy-trend variable to equation (26) to proxy the broad policy-related pattern of real rates. The estimated equation for real rates becomes,

$$p_{nt}^{R} = G_n + H_n w_t + \psi D_t \tag{30}$$

This full-sample variable, D_t, takes the period-specific value of the mean of the Bank's 10-year real rate in the each of the periods: 1983Q2-1989Q4, 1990Q1-1992Q3[11], 1997Q4 to 2004Q4; and follows a linear trend in the period

[10] See http://www.bankofengland.co.uk/statistics/yieldcurve/index.htm.

[11] This period begins before the start of the of ERM regime because sterling was 'shadowing' the Deutschmark before joining the Mechanism

1992Q3-1997Q4.[12]

5 Empirical Results.

5.1 Maximum Likelihood Estimates of β and γ

Estimates of the coefficients β and γ are presented in Table 1, along with likelihood values, and t-statistics in parentheses. The traditional representative agent model performs reasonably well: The estimates of β are sensible (implying a discount rate of about 1.5%) and highly significant; the estimates of γ are less precise but generally positive. The estimates for INC and PIPO are slightly better, with positive estimates of γ throughout, ranging from 0.12 to 0.93 for the no-insurance model, and from 0.17 to 1.062 for the partial-insurance model. The likelihood values are marginally higher for the INC and PIPO models. While the differences do not allow us to conclude that the limited-insurance models significantly outperform the full-insurance model, it is clear that introducing the insurance issue does not lead to a deterioration in the models' performance.

We measure the goodness of fit using Nagelkerke's (1991) generalized R^2,

$$R^2 = 1 - \left(\frac{L(0,0)}{L(\hat{\beta},\hat{\gamma})} \right)^{\frac{2}{n}}$$

where n is the number of bonds in the sample, and L is the value of the likelihood function.

The results are very similar for each model but they vary significantly across samples. The models explain about 7% of the cross-section and time-series variation in bond prices in the full sample, but explain rather more than this when the samples are limited according to monetary policy regimes. The strongest performance arises during the short ERM period, when all of the models explain about 50% of the price variation. This drops to about 22% for the period of Bank of England independence.

[12] Estimates of ψ proved to be statistically significant in all of our estimation samples with the exception of the ERM period, for which D_t is a constant.

5.2 Risk premia.

The expected 1-period return on an n-period pure real bond is

$$E_t[r_{n,t+1}] = -E_t[m_{t+1}] - \frac{1}{2}Var_t[m_{t+1} + p_{n-1,t+1}] \tag{31}$$

from which we get the expected excess return ($\tilde{r}$) of an n-period bond over that of an s-period bond as

$$\begin{aligned} E_t[\tilde{r}_{n,s,t+1}] &= -\frac{1}{2}(Var_t[p_{n-1,t+1}] - Var_t[p_{s-1,t+1}]) - \\ &\qquad (Cov_t[m_{t+1}, p_{n-1,t+1}] - Cov_t[m_{t+1}, p_{s-1,t+1}]) \\ &= -\frac{1}{2}(H_{n-1} - H_{s-1})\Omega_\epsilon(H_{n-1} - H_{s-1})' - (H_{n-1} - H_{s-1})\Omega_\epsilon\phi_R \end{aligned}$$

In the case of a one-period, and therefore riskless, bond we have $H_{s-1} = H_0 = 0$ from which we get the *ex ante* risk premium on an n period bond as

$$E_t[\tilde{r}_{n,1,t+1}] \equiv \rho_n = -\frac{1}{2}H_{n-1}\Omega_\epsilon H'_{n-1} - H_{n-1}\Omega_\epsilon\phi_R \tag{32}$$

Our assumption that the errors in the VAR are homoskedastic (i.e. that Ω_ϵ is constant) results in *ex ante* risk premia that are time invariant. Table 2 shows that the point estimates of these *ex-ante* risk premia are small, ranging from -0.02% to +0.15% p.a, depending on the model and the sample period. Only the model in which markets fail to provide any insurance produces positive risk premia in all periods.[13]

The low estimated risk aversion parameter, and the resulting small implied *ex-ante* risk premia raises the question of why investors appear to demand so little compensation for term-structure risks. The poor returns on indexed bonds are noted in Dimson *et al.* (2002) who find that indexed bonds returned 1.25% p.a. less than Treasury bills for the period 1981 to 2000. They offer 2 possible explanations:

> '...their poor performance stems from unexpected increases in the real rate of interest particularly in the early 1980s.'

and

[13] Campbell and Viceira (2002), in a study using only nominal bonds and equities, report real risk premia of between 1% and 2% for the US market.

> 'Since most inflation-indexed bonds have low coupons, and since there is no capital gains tax on UK government bonds, they are attractive to high-rate taxpayers relative to most conventional bonds. The low returns on inflation-indexed bonds may therefore also partly reflect the influence of tax clienteles' (Dimson *et al* (2002), page 86.)

Our results suggest a third possibility i.e. that the low returns may have arisen from a low aversion to risk on the part of indexed-bond investors. This explanation may complement Dimson *et al.*'s tax-clienteles argument. It also suggests that, while unexpected increases in real rates may be part of the explanation, they are not a necessary part since low returns could arise from low risk aversion even if interest expectations were, on average, correct.

5.3 The response of real interest rates to factor shocks.

5.3.1 Impulse effects.

We examine the impulse responses of real interest rates to shocks to the factors in the form of 1-period ahead expectations of consumption growth, the change in cross-sectional consumption variance, and inflation.

The system of equations can be represented as,

$$
\begin{aligned}
p^R_{n,t} &= G_n + H_n w_t \quad n = 1, 2, ... && (33) \\
w_t &= A + B w_{t-1} + \epsilon_t && (34)
\end{aligned}
$$

from which we get the prices as functions of the history of the factor shocks ϵ as

$$p^R_{n,t} = G_n + H_n (I - B)^{-1} A + H_n (I - BL)^{-1} \epsilon_t \qquad (35)$$

Real interest rates at all maturities n follow directly from this equation.

The ϵ_t terms are mutually correlated so we recast these as linear functions of three orthogonal random terms ξ_t and measure the response of real rates to shocks to the latter. Thus we assume that,

$$
\begin{aligned}
\epsilon_{1t} &= c_{11}\xi_{1t} + c_{12}\xi_{2t} + c_{13}\xi_{3t} && (36)\\
\epsilon_{2t} &= c_{21}\xi_{2t} + c_{22}\xi_{2t} + c_{23}\xi_{3t} && (37)\\
\epsilon_{3t} &= c_{31}\xi_{3t} + c_{32}\xi_{2t} + c_{33}\xi_{3t} && (38)
\end{aligned}
$$

This leads to the familiar problem that we cannot identify all 9 c_{ij} coefficients from the 6 independent coefficient estimates in $\hat{\Omega}_\epsilon$. We deal with this in the usual way with a Cholesky decomposition of the covariance matrix Ω_ϵ i.e. we impose zero-restrictions on c_{12}, c_{13} and c_{23}. This is equivalent to assuming that the shock ξ_{1t} influences all three variables, ξ_{2t} influences only the latter two, and ξ_{3t} influences only the third. Since the ordering of the ϵs is not unique (we could put the 3 variables in the VAR in any order), and because we have no prior information as to the real-world ordering under these identifying restrictions (if in fact any is correct), we present results for four of the six possible orderings; for the remaining two the reordered Ω is not positive definite and, therefore, there is no Cholesky decomposition.

Thus we define

$$
\begin{aligned}
\epsilon_t &= C\xi_t && (39)\\
E[\xi_t\xi_t'] &= I && (40)
\end{aligned}
$$

where C is lower-triangular

Hence,

$$\Omega_\epsilon = CC' \qquad (41)$$

Substituting (39) into the bond price equations we get:

$$p^R_{n,t} = G_n + H_n(I-B)^{-1}A + H_n(I-BL)^{-1}C\xi_t \quad n = 1, 2, ... \qquad (42)$$

In order to give the shocks to the ξ a clearer economic meaning we scale them such that they generate a 1 percentage point increase in each of the factors in turn. For example, in Table 3, the first row shows the effects of a shock to ξ_{1t} such that consumption growth increases by 1%. In line with the ordering of the VAR, this same ξ_{1t} shock also generates a contemporaneous

30.71% increase in the change of the cross-sectional variance of consumption, and a 0.12% decline in inflation. for our estimated models, the qualitative effects on yields of all of the factor shocks turn out to be robust to changes in the order of the factors.

The effect of a shock to inflation that does not cause contemporaneous shocks to the other two factors can be seen from lines 3 and 6 of Table 3. For both the INC and PIPO models, there is no impact on the 1-period (i.e. 3-month) real rate, but there are small falls in real rates at longer maturities. This is as expected: the short real rate does not respond to changes in expected inflation since agents are assumed to optimize their utility over real magnitudes, and there are no changes in the consumption factors in the utility function. At longer maturities however, the effect of a current inflation shock on expectations of future consumption growth and variance do have an impact on real rates by altering the utility value of future real returns. the negative response of longer real rates is consistent with results found in Barr and Campbell (1997) and others, and provide a possible explanation for their results. This negative impact of expected inflation on real rates arises for all of the VAR orderings, although with inflation placed at position 1 or 2 in the VAR the associated contemporaneous shocks to the other factors generate negative responses in the 3-month rate also.

Increases in consumption growth lead to increases in real rates for both models (with the exception of the 2-year rates for the INC model) irrespective of the ordering of the factors in the VAR: higher consumption growth lowers agent's incentive to save, and financial markets respond by offering a higher real yield as the demand for real bonds declines.

Positive shocks to the cross-sectional variance of consumption cause real rates to fall in all cases, which is consistent with the Euler equations (2) and (3) given that the estimated $\gamma < 1$ in both the INC and PIPO models. A higher cross-sectional variance in consumption, means that consumers face greater uninsurable risk. In both INC and PIPO environments with zero or partial insurance, consumers increase their saving for precautionary reasons and this increase in the supply of loanable funds drives down real interest rates. In a PIPO environment, with partial insurance, an opposing effect on saving will be at work. Greater consumption inequality (i.e. a higher cross sectional consumption variance) lowers the agency cost because it becomes cheaper to provide incentives to poorer households. This lower agency cost may create a wealth effect that lowers the incentive to save. Thus we expect that the effect on real rate will be weaker in the PIPO environment.

To summarise the main results: A positive shock to inflation lowers real rates as a consequence of its effects on consumption growth and consumption variance; a positive shock to consumption growth raises real interest rates as markets compete for reduced savings, and a positive shock to uninsurable consumption risk lowers real rates as the precautionary motive drives agents to save more.

6 Summary and conclusions

This paper tests three consumption-based asset pricing models applied to a combined cross-section/time-series sample of indexed bond prices in the UK. We employ a three factor model of log normal bond pricing and derive closed form expressions for the pricing kernels of the new class of uninsurable risk models. This innovation allows us to derive the price function of indexed coupon bonds in an estimable form with a convenient marriage between VAR based representation of the state variables and the bond price equation. Our central equation is a log linear bond price equation in which expected values of the state variables are constructed from a parsimonious VAR involving three macroeconomic variables, namely the growth rate of aggregate consumption, cross-section variance of consumption and the rate of inflation.

We find that the standard complete markets model with homogenous agents performs reasonably well with inflation-indexed bonds, in contrast to its typical performance in equity models. Our results lend slightly more support to the new class of uninsurable risk models, which marginally outperform the standard model. The models explain up to 50% of the (detrended) variation in bond prices. The impulse response analysis of the estimated bond price equations reveals that a rise in inflation temporarily lowers real interest rates at all maturities (with the exception of the 'safe' maturity of 3 months) while a rise in aggregate consumption growth rate raises real interest rates. An increase in uninsurable risk, on the other hand, lowers real rates.

Our results raise some questions for future research. For example, why are the estimated coefficient of relative risk aversion and the resulting bond risk premia smaller in the UK indexed bond market than is typically the case for equity markets? One possible explanation is that UK bond market is extremely segmented and populated with near risk neutral institutional investors. An alternative explanation may be that our utility function could be misspecified. A more general function, combining the uninsurable risk features discussed here, with the separation of risk aversion and intertemporal substitution in consumption as in Epstein and Zin (1991), may lead to larger estimates of the degree of risk aversion. To the best of our knowledge however, there is as yet no theory that integrates these incomplete market models with non-expected utility maximization, and is likely to be a productive avenue for further research.

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A Construction of the Cross Sectional Distribution of Consumption

We construct the cross sectional variance of real consumption using the records of daily expenditure from the Family Expenditure Survey (FES) conducted by the Office for National Statistics (ONS). The data we use are based on the expenditure of approximately 6,500 households for a period of 2 weeks in every quarter.

Our procedure mimics that of Kocherlakota and Pistaferri (2007). First, the household-wide consumption of nondurables and services is calculated by adding the nominal consumption of nondurables and services for each individual in the household. We follow the definition of nondurable and services of Attanasio and Weber (1995). Second, since the household consumption data are two week durations only, we multiply them by 6.5 to arrive at quarterly frequency. Third, we divide this quarterly consumption expenditure of each household by the number of people in the household in that quarter to arrive at the quarterly nominal, consumption of nondurables and services per member of each household unit. Fourth, by dividing the quarterly data by the quarterly CPI for all items (not seasonally adjusted) (the CPI is from the OECD main economic indicators) with the basis of 2005:Q1, we arrive at the quarterly real per capita consumption for all the relevant households.

A.1 Measurement errors

KP (2009) raise the issue of measurement errors that arise from from the use of cross-section expenditure data. In our context, if these measurement errors appear multiplicatively they do not impact the pricing kernels. To see this define the measured consumption as:

$$\hat{c}_{i,t} = c_{i,t} \exp(\xi_{i,t}) \tag{A.1}$$

where the measurement error $\xi_{i,t}$ is stationary, i.i.d. across households, and uncorrected with z_t, we get:

$$\hat{x}_t - \hat{x}_{t-1} = x_t - x_{t-1} \tag{A.2}$$

Since we work with the first difference of the variance of log consumption, the measurement error is not an issue.

B Derivation of the estimated price equations.

B.1 From zero-coupon to coupon bonds.

We price a coupon bond as the sum of the prices of its coupons and redemption payment. I.e.

$$P_{nt}^{N,C} = \sum_{s=1}^{n} P_{st}^{Nom} C + P_{nt}^{Nom} \tag{B.3}$$

hence, since

$$P_{st}^{Nom} = exp(p_{st}^{Nom}) \tag{B.4}$$

we have the log price of a coupon bond as

$$P_{nt}^{N,c} = \sum_{s=1}^{n} exp(p_{st}^{Nom})C + exp(p_{nt}^{Nom}) \tag{B.5}$$

B.2 Incorporating the macroeconomic factors.

First, we introduce a vector notation for the factors i.e.

$$w_t = \begin{pmatrix} g_t \\ v_t \\ \pi_t \end{pmatrix} \tag{B.6}$$

where $g_t = c_t - c_{t-1}$, $v_t = x_t - x_{t-1}$ and $\pi_t = q_t - q_{t-1}$ and q_t is the RPI.

Now introduce 2 selection vectors to allow us to pick out different combinations of the factors i.e.

$$\phi_R = \begin{pmatrix} -\gamma \\ \pm\left(\frac{\gamma(\gamma\pm1)}{2}\right) \\ 0 \end{pmatrix} \tag{B.7}$$

$$\phi_L = \begin{pmatrix} -\gamma \\ \pm\left(\frac{\gamma(\gamma\pm1)}{2}\right) \\ \text{-1} \end{pmatrix} \tag{B.8}$$

B.3 The stochastic discount factors.

For the stochastic discount factors (12) and (13),

$$\begin{aligned}
M_{t+i} &= \beta G_{t+i}^{-\gamma} \exp\left[\left(\frac{\gamma(\gamma\pm1)}{2}\right)(x_{t+i} - x_{t+i-1})\right] && \text{(B.9)}\\
\Rightarrow m_{t+i} &= \ln\beta - \gamma g_{t+i} \pm \left[\left(\frac{\gamma(\gamma\pm1)}{2}\right)(x_{t+i} - x_{t+i-1})\right] && \text{(B.10)}\\
&= \ln\beta - \gamma g_{t+i} \pm \left[\left(\frac{\gamma(\gamma\pm1)}{2}\right) v_{t+i})\right] && \text{(B.11)}\\
&= \ln\beta + \phi_R' w_{t+i} && \text{(B.12)}
\end{aligned}$$

Now substitute this expression for m into the price equation (16) to get,

$$\begin{aligned}
p_{nt}^{Nom} - (q_t - q^*) &= E_t\left(\phi_R' w_{t+1} + ... + \phi_R' w_{t+n-3} + \phi_L' w_{t+n-2} + \phi_L' w_{t+n-1} + \phi_L' w_{t+n}\right) + \\
&\quad \frac{1}{2} Var_t(\phi_R' w_{t+1} + ... + \phi_R' w_{t+n-3} + \phi_L' w_{t+n-2} + \phi_L' w_{t+n-1} + \phi_L' w_{t+n}) && \text{(B.13)}
\end{aligned}$$

The terms $\phi_R' w_{t+1} + ... + \phi_R' w_{t+n-3}$ come directly from the equation for m above. The others, $\phi_L' w_{t+n-2} + \phi_L' w_{t+n-1} + \phi_L' w_{t+n}$, are a combination of the m terms and inflation, for the last 3 months of the bond's life i.e. the period after the indexation ends, and the bond's real value is exposed to inflation.

So a convenient alternative way to write z is,

$$z_{n,t+1} = (\phi_R' w_{t+1} + ... + \phi_R' w_{t+n-3} + \phi_L' w_{t+n-2} + \phi_L' w_{t+n-1} + \phi_L' w_{t+n}) \tag{B.14}$$

B.4 Time series projections for the factors.

For the case of a VAR(1) (and for higher-order VARs in companion form) we have,

$$\begin{aligned} w_{t+1} &= A + Bw_t + \epsilon_{t+1} \\ &= A_n + B_n w_t + \eta_{t+n} \end{aligned} \tag{B.15}$$

where

$$A_n = (I + B + ... + B^{n-1})A \tag{B.16}$$
$$B_n = B^n \tag{B.17}$$
$$\eta_{t+n} = \epsilon_{t+n} + B\epsilon_{t+n-1} + ... + B^{n-1}\epsilon_{t+1} \tag{B.18}$$

It follows that, introducing $\Omega_{t+n} \equiv Var_t(\eta_{t+n})$, which we assume to be constant w.r.t t,

$$E_t(w_{t+n}) = A_n + B_n w_t \tag{B.19}$$
$$\begin{aligned} Var_t(w_{t+n}) &= Var_t(\eta_{t+n}) \\ &= \Omega + B_1\Omega B_1' + ... + B_{n-1}\Omega B_{n-1}' \\ &= \Omega_{t+n} \end{aligned} \tag{B.20}$$

More compactly,

$$\Omega_{t+i} = \sum_{j=0}^{i-1} B_j \Omega_\epsilon B_j' \quad \forall t, \text{ and } i = 1...n \tag{B.21}$$

B.5 Derivations of $E_t(z)$ and $Var_t(z)$.

Given that

$$z_{n,t+1} = +\left(\phi_R' w_{t+1} + ... + \phi_R' w_{t+n-3} + \phi_L' w_{t+n-2} + \phi_L' w_{t+n-1} + \phi_L' w_{t+n}\right) \tag{B.22}$$

we get,

$$
\begin{aligned}
E_t(z_{n,t+1}) &= E_t\left[\phi_R'\left(w_{t+1}+...+w_{t+n-3}\right)+\phi_L'\left(w_{t+n-2}+w_{t+n-1}+w_{t+n}\right)\right] \\
&= \phi_R'\left((A_1+B_1w_t)+...+(A_{n-3}+B_{n-3}w_t)\right)+ \\
&\quad \phi_L'\left((A_{n-2}+B_{n-2}w_t)+(A_{n-1}+B_{n-1}w_t)+(A_n+B_nw_t)\right) \\
&= \phi_R'\left(A_1+...+A_{n-3}\right)+\phi_L'\left(A_{n-2}+A_{n-1}+A_n\right)+ \\
&\quad \phi_R'\left(B_1w_t+...+B_{n-3}w_t\right)+\phi_L'\left(B_{n-2}w_t+B_{n-1}w_t+B_nw_t\right) \\
&= \phi_R'\left(A_1+...+A_{n-3}\right)+\phi_L'\left(A_{n-2}+A_{n-1}+A_n\right)+ \\
&\quad \phi_R'\left(B_1+...+B_{n-3}\right)w_t+\phi_L'\left(B_{n-2}+B_{n-1}+B_n\right)w_t \\
&= \left[\phi_R'\left(A_1+...+A_{n-3}\right)+\phi_L'\left(A_{n-2}+A_{n-1}+A_n\right)\right]+ \\
&\quad \left[\phi_R'\left(B_1+...+B_{n-3}\right)+\phi_L'\left(B_{n-2}+B_{n-1}+B_n\right)\right]w_t \\
&= \left(\sum_{i=1}^{n-3}\phi_R'A_i+\sum_{i=n-2}^{n}\phi_L'A_i\right)+ \\
&\qquad \left(\sum_{i=1}^{n-3}\phi_R'B_i+\sum_{i=n-2}^{n}\phi_L'B_i\right)w_t
\end{aligned} \tag{B.23}
$$

$$
\begin{aligned}
Var_t(z_{n,t+1}) &= \phi_R'\left(\Omega_{t+1}+...+\Omega_{t+n-3}\right)\phi_R+\phi_L'\left(\Omega_{t+n-2}+...\Omega_{t+n}\right)\phi_L \\
&= \phi_R'\left(\sum_{j=0}^{0}B_j\Omega_\epsilon B_j'+...+\sum_{j=0}^{n-3-1}B_j\Omega_\epsilon B_j'\right)\phi_R+ \\
&\quad \phi_L'\left(\sum_{j=0}^{n-2-1}B_j\Omega_\epsilon B_j'+...\sum_{j=0}^{n-1}B_j\Omega_\epsilon B_j'\right)\phi_L
\end{aligned} \tag{B.24}
$$

B.6 The final equation for a zero-coupon indexed bond.

Recall,

$$p_{nt}^{Nom} - (q_t - q^*) = E_t(z_{n,t+1}) + \frac{1}{2}Var_t(z_{n,t+1}) \tag{B.25}$$

we can substitute for the conditional expectations and variances of w that appear in z. The expectations introduce a series of terms in the constant A, which when added to the constant conditional variance, gives us the constant term in the price equation.

The time-varying element i.e. the terms in the factors w_{t+i} are all functions of w_t. Hence, the real price, $p_{nt}^R \equiv p_{nt}^{Nom} - (q_t - q^*)$, is

$$p_{nt}^R = G_n + H_n w_t \tag{B.26}$$

where

$$\begin{aligned} G_n &= ln(\beta) + \left(\sum_{i=1}^{n-3} \phi_R' A_i + \sum_{i=n-2}^{n} \phi_L' A_i \right) + \qquad \text{(B.27)} \\ &\quad \frac{1}{2} \left(\sum_{i=1}^{n-3} \sum_{j=0}^{i-1} \left(\phi_R' B_j \Omega_\epsilon B_j' \phi_R \right) + \sum_{i=n-2}^{n} \sum_{j=0}^{i-1} \left(\phi_L' B_j \Omega_\epsilon B_j' \phi_L \right) \right) \\ H_n &= \sum_{i=1}^{n-3} \phi_R' B_i + \sum_{i=n-2}^{n} \phi_L' B_i \qquad \text{(B.28)} \end{aligned}$$

Table 1: Estimation results.

	RA			INC			PIPO		
	β	γ	[R^2] LF	β	γ	[R^2] LF	β	γ	[R^2] LF
All	0.9928	0.1155	[0.07420]	0.9933	0.2516	[0.07424]	0.9936	0.2742	[0.07423]
	(810.2)	(0.6654)	10.89	(1360.)	(1.720)	10.90	(940.9)	(1.672)	10.90
Pre-ERM	0.9901	-0.1039	[0.2079]	0.9932	0.4728	[0.2083]	0.9919	0.1690	[0.2080]
	(564.4)	(-0.4172)	12.05	(1427.)	(2.686)	12.09	(1097.)	(1.164)	12.05
ERM	0.9913	0.2894	[0.5329]	0.9928	0.9346	[0.5393]	0.9922	0.4716	[0.5345]
	(335.8)	(0.6603)	12.01	(4186.)	(3.987)	12.35	(729.4)	(2.337)	12.10
Pre-BoE	0.9966	0.8237	[0.3439]	0.9928	0.3965	[0.3456]	0.9951	0.6554	[0.3459]
	(263.3)	(1.547)	11.92	(2584.)	(4.536)	12.03	(908.9)	(4.529)	12.05
BoE	0.9939	0.9440	[0.2231]	0.9884	0.2140	[0.2221]	0.9934	0.8974	[0.2232]
	(322.7)	(2.263)	13.43	(950.4)	(1.242)	13.31	(627.7)	(5.071)	13.44
Phase 1	0.9953	0.2710	[0.1602]	0.9958	0.5084	[0.1606]	0.9962	0.4297	[0.1604]
	(534.2)	(1.135)	11.88	(1966.)	(4.096)	11.94	(815.5)	(2.645)	11.92
Phase 2	0.9951	0.6067	[0.2560]	0.9934	0.5674	[0.2591]	0.9945	0.5675	[0.2576]
	(255.7)	(1.112)	11.34	(3427.)	(6.011)	11.60	(1285.)	(5.579)	11.48
Phase 3	1.003	1.150	[0.2599]	0.9957	0.1219	[0.2579]	1.003	1.062	[0.2599]
	(320.1)	(2.637)	13.08	(1079.)	(0.7249)	12.89	(667.5)	(6.195)	13.08

LF is the log likelihood value; t-statistics in parentheses. R^2 (in square brackets) is Nagelkerke's (1991) measure.

Table 2: Implied *ex-ante* risk premia (% p.a).

	RA	INC	PIPO
All	-0.0003601	0.003962	0.0006149
Pre-ERM	-0.0002916	0.02073	0.0004890
ERM	-0.002262	0.1481	-0.001330
Pre-BoE	-0.01833	0.01287	-0.006906
BoE	-0.02407	0.002652	-0.02001
Phase 1	-0.001983	0.02533	-0.0006021
Phase 2	-0.009944	0.03443	-0.003743
Phase 3	-0.03570	0.0006971	-0.03146

Table 3: Impulse responses to factor shocks.

Cholesky order	Factor shocks			Yield response: INC			Yield response: PIPO		
	Growth	Variance.	Inflation	3 month	2 year	10 year	3 month	2 year	10 year
	1.00	30.7	-0.120	0.0570	0.0215	0.00440	0.842	0.186	0.0381
g v π	0.000	1.00	7.70e-005	-0.00511	-0.000720	-0.000144	-0.00181	-0.000410	-8.36e-005
	0.000	0.000	1.00	0.000	-0.00125	-0.000333	0.000	-0.0527	-0.0112
	0.00412	1.00	-0.000249	-0.0160	-0.00220	-0.000441	-0.00230	-0.000540	-0.000109
v g π	1.00	0.000	-0.122	0.214	0.0437	0.00883	0.897	0.199	0.0407
	0.000	0.000	1.00	0.000	-0.00125	-0.000333	0.000	-0.0527	-0.0112
	1.00	8.12	-0.120	0.0570	0.0215	0.00440	0.842	0.186	0.0381
g π v	0.000	0.389	1.00	-0.00753	-0.000720	-0.000546	-0.00267	-0.0533	-0.0114
	0.000	1.00	0.000	-0.0193	-0.00125	-0.000545	-0.00685	-0.00154	-0.000313
	-0.0957	-1.48	1.00	-0.00485	-0.00220	-0.000961	-0.0832	-0.0705	-0.0149
π v g	0.00124	1.00	0.000	-0.00485	0.0437	-0.000133	-0.000702	-0.000168	-3.41e-005
	1.00	0.000	0.000	0.214	-0.00125	0.00879	0.897	0.192	0.0393